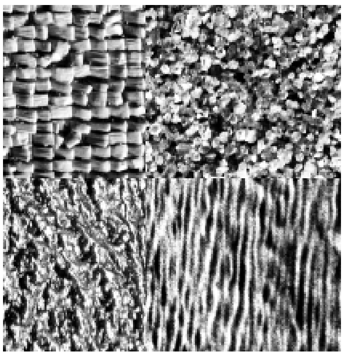
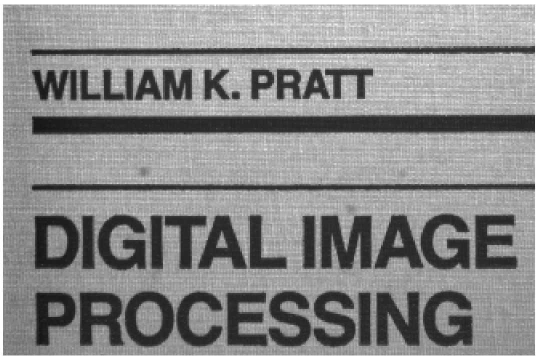
7 – EDGE DETECTION

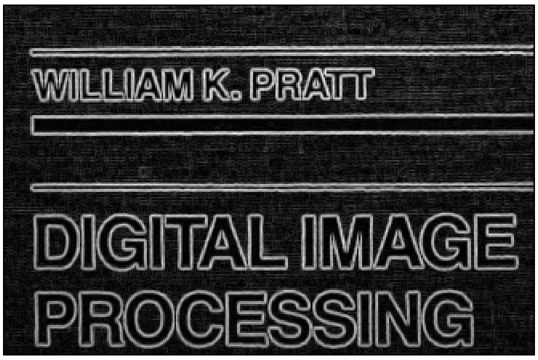
Changes or discontinuities in an image attribute such as luminance or texture are important primitive features of an image since they often provide an indication of the physical extent of objects within the image. Edge detection contributes significantly to algorithms for feature detection, segmentation, and motion analysis.

In this course, we consider edges to be local discontinuities in intensity.

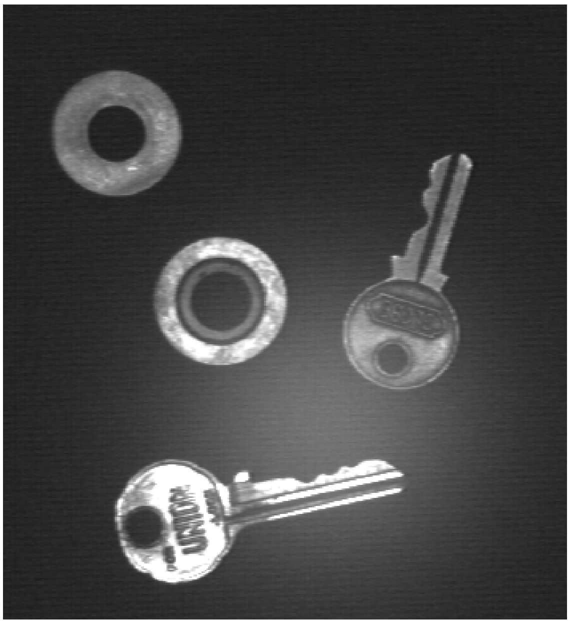


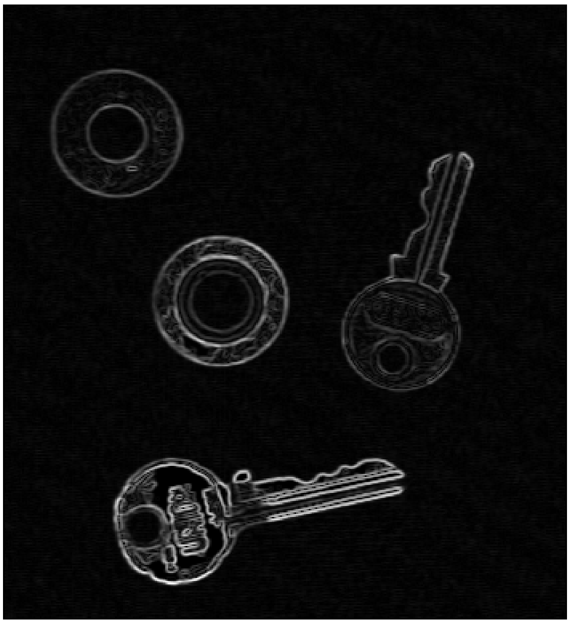


Original



Detected edges


Original

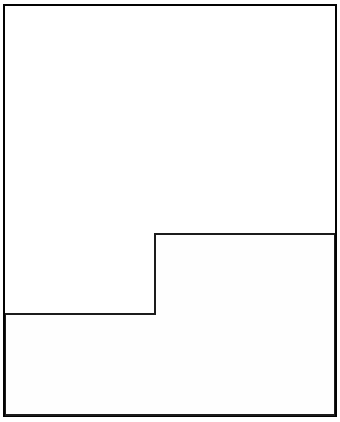

Detected edges

Original

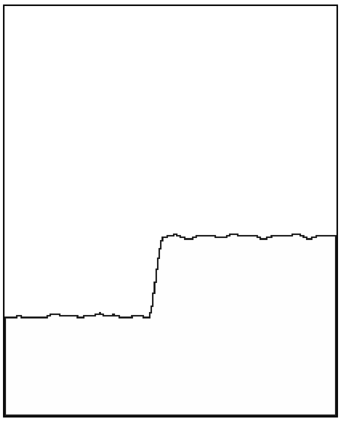
Detected edges

Threshold and invert

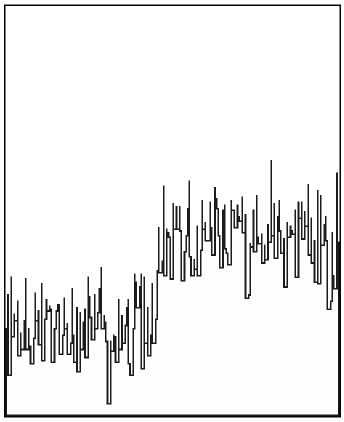
Example - Edge profiles



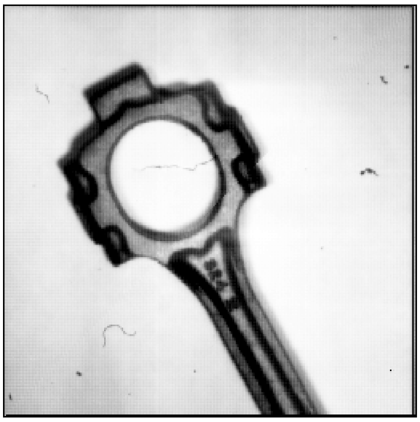
Ideal edge

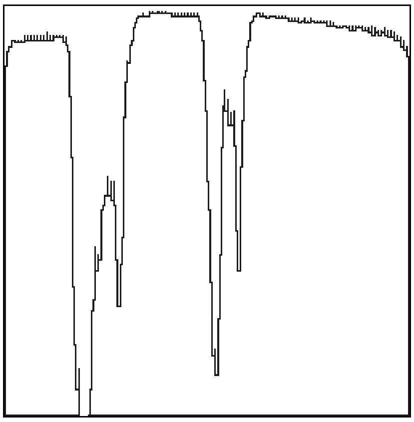


"Real" edge



Noisy edge





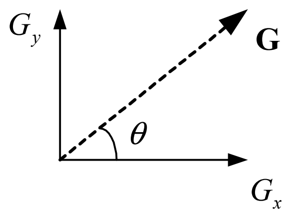
Gradient Operators

For an image function $f(x, y)$, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are the rates of change in the x and y directions, respectively. The gradient at any point (x, y) is defined as

$$\mathbf{G}(x, y) = \begin{bmatrix} \partial f(x, y)/\partial x \\ \partial f(x, y)/\partial y \end{bmatrix}. \tag{1}$$

The direction of the gradient vector $\mathbf{G}$ at (x, y) measured with respect to the x axis is

$$\theta(x, y) = \tan^{-1}(G_y/G_x) \tag{2}$$



where

$$G_x = \frac{\partial f}{\partial x}, \qquad G_y = \frac{\partial f}{\partial y}$$

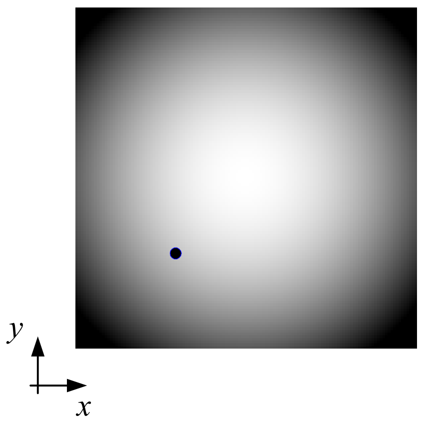
The gradient vector points in the direction of maximum rate of change of f at location (x, y).

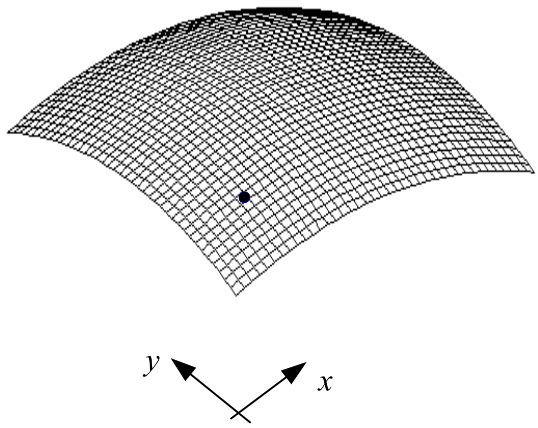
For edge detection, we are interested in the magnitude of the gradient:

$$|\mathbf{G}(x, y)| = [G_x^2 + G_y^2]^{1/2}. \tag{3}$$

An approximation is

$$|\mathbf{G}(x, y)| \approx |G_x| + |G_y|. \tag{4}$$





How do we obtain the gradient at any point in a digital image?

Using two-point approximations:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} \qquad (5)$$

$$\frac{\partial f(x,y)}{\partial y} \approx \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} \qquad (6)$$

which correspond to a masking operation with

$$[-1 \quad 1] \qquad \text{and} \qquad \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

(Δx and Δy are left as overall window weights, x axis points to the right, y axis points up.)

Along one dimension, we define the forward derivative approximation, D_1, as

$$D_{1x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} \qquad (7)$$

Better estimates are obtained using the *centred* difference approximation

$$D_{2x} = \frac{f(x+\Delta x) - f(x-\Delta x)}{2(\Delta x)} \qquad (8)$$

equivalent to masking operations with

$$[-1 \quad 0 \quad 1] \qquad \text{and} \qquad \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

Variants on D_2 are the Prewitt masks:

$$D_{Px} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D_{Py} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

which form smoothed, centred difference operators.

The *Sobel* weighting masks emphasize the central pixel:

$$D_{Sx} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D_{Sy} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

The Sobel edge filter provides good edge detection and is somewhat insensitive to noise in the image.

The *Roberts* operator is defined by

$$D_+ = f(x + \Delta x, y + \Delta y) - f(x, y) \tag{9}$$

$$D_- = f(x, y + \Delta y) - f(x + \Delta x, y) \tag{10}$$

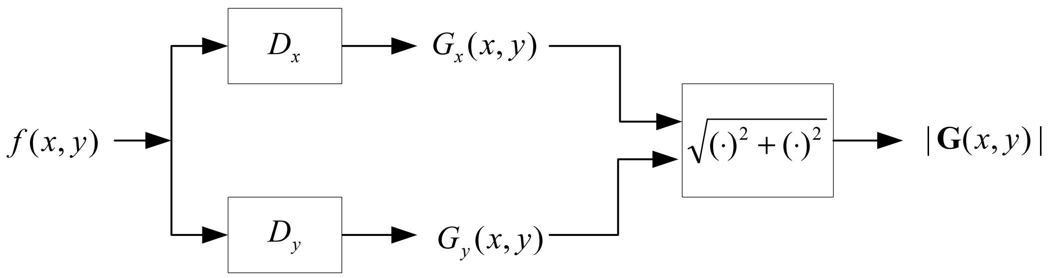
corresponding to filtering the image function with masks

$$D_+ = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad D_- = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The centre of each mask may be taken to be the lower left pixel.

The output image $g(x, y)$ can be set equal to the gradient of the input image $f(x, y)$ at that point:

$$g(x, y) = |\mathbf{G}[(x, y)]|. \tag{11}$$



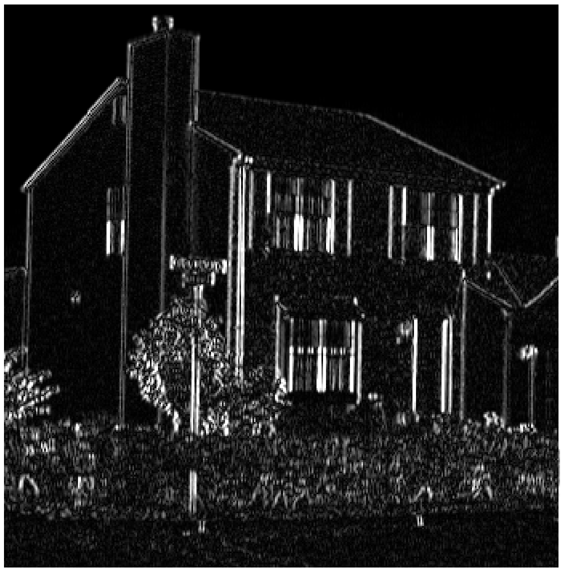
To obtain an edge map:

$$g(x, y) = \begin{cases} 1 & \text{if } |\mathbf{G}[(x, y)]| > T \\ 0 & \text{if } |\mathbf{G}[(x, y)]| \leq T. \end{cases} \tag{12}$$

The edge map gives the necessary data for tracing the object boundaries in an image. Typically, T may be selected using the cumulative histogram of $G(x, y)$ so that 5 to 10% of pixels with the largest gradients are declared as edges.

Edge detection

Original



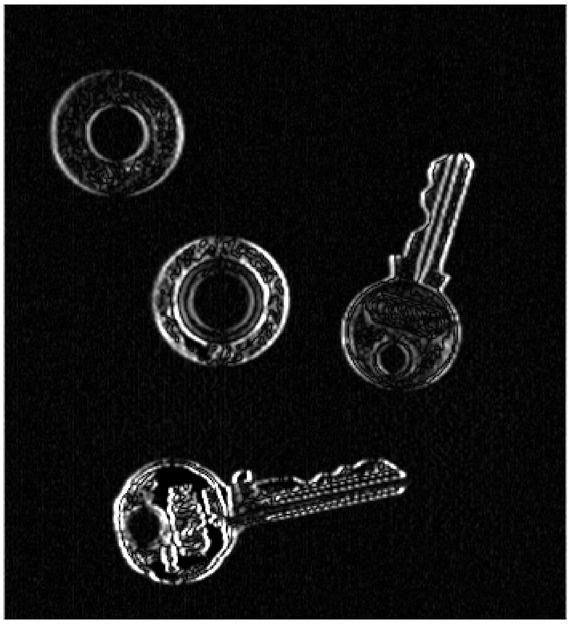
G_x

G_y

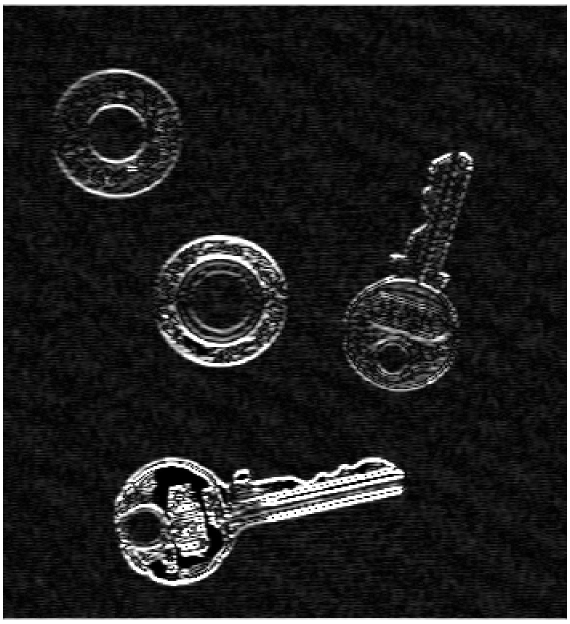
$|\mathbf{G}(x, y)|$

Edge detection

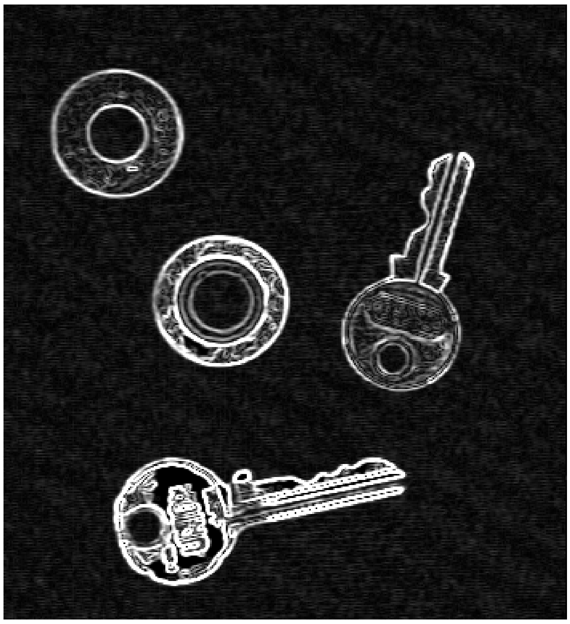
Original



G_x



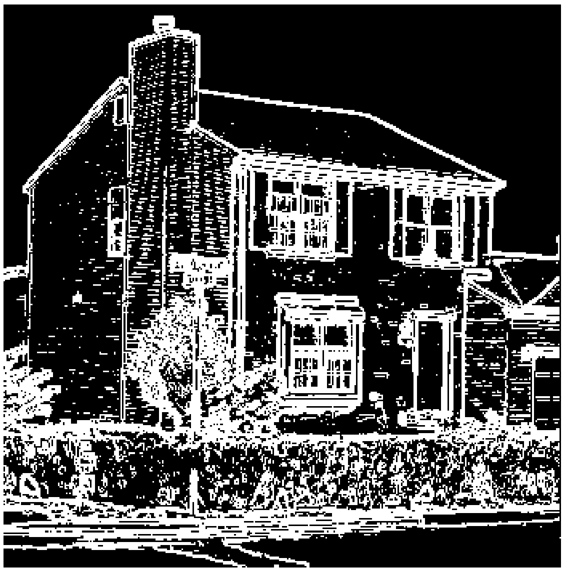
G_y



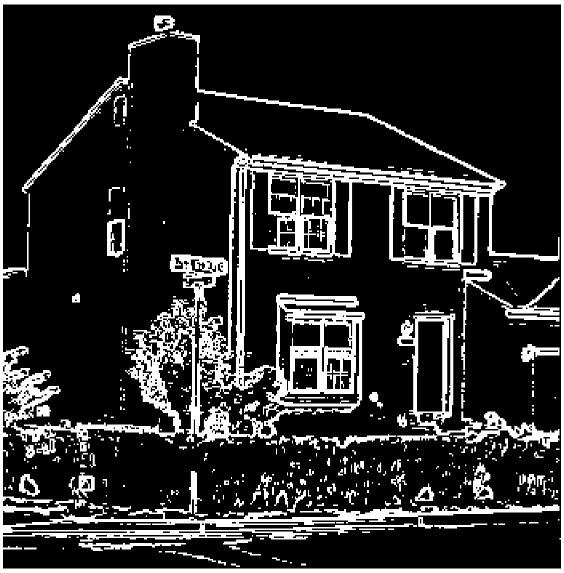
$|\mathbf{G}(x,y)|$

Edge map

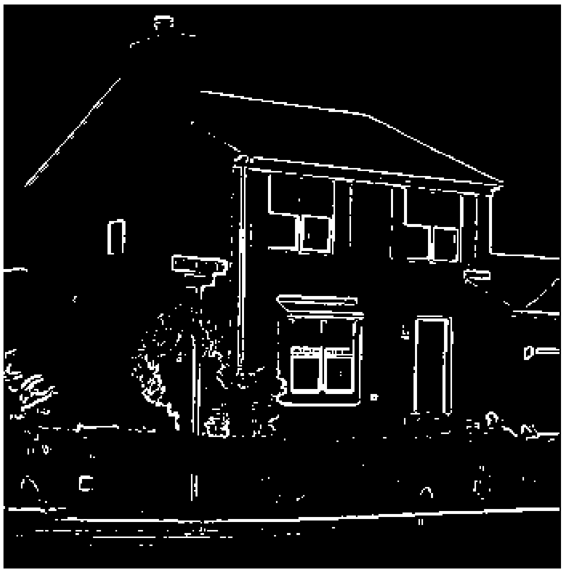
Edge magnitude



With low threshold



With moderate threshold



With high threshold

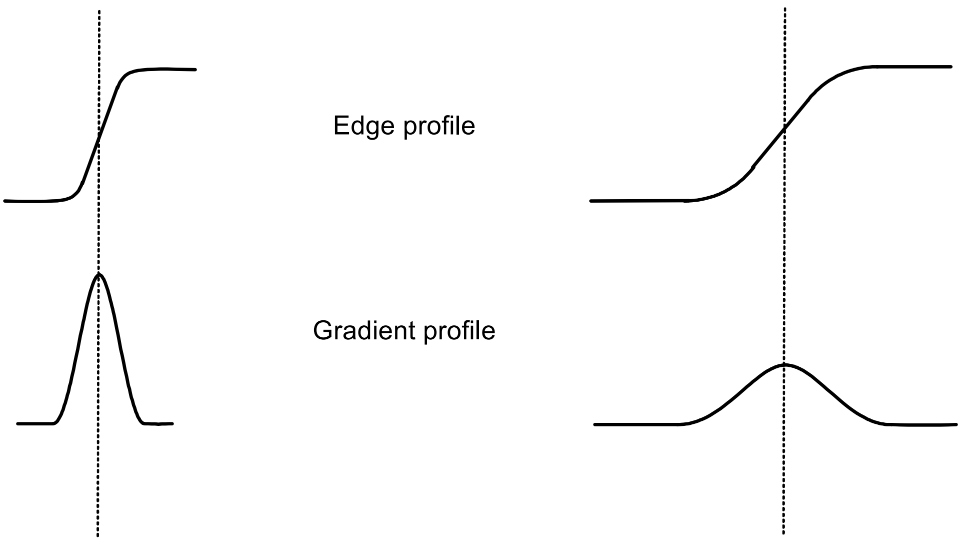
Problems

Problems may be encountered with the use of gradient operators:

1. The boundaries may be diffused instead of sharp; the former have lower gradients than the latter, thus complicating the threshold selection.

Slightly blurred rectangle

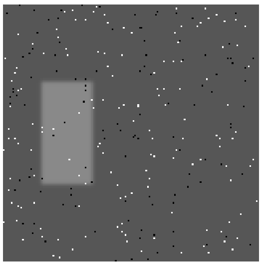
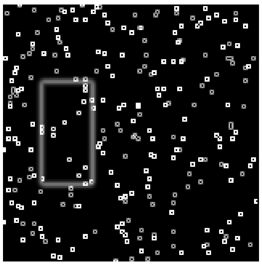
Heavily blurred rectangle



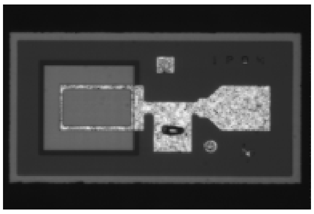
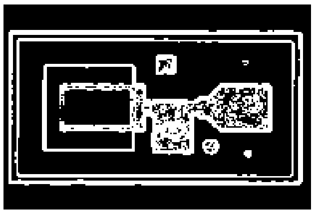
Profiles across one edge of the rectangle

Profiles across one edge of the rectangle

2. Noise can result in stronger gradients than meaningful edges.

3. Edges after thresholding are often thick.

More sophisticated algorithms (e.g., Canny operator) can be used to give better results.

Laplacian Operator

We have from Eq. (7) the forward difference:

$$D_1(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}. \tag{13}$$

Similarly, the "backward" difference is

$$D_{-1}(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}. \tag{14}$$

The second difference (analogous to the second derivative) is then

$$D_1^2(x) = \frac{1}{\Delta x}\left[\frac{f(x+\Delta x) - f(x)}{\Delta x} - \frac{f(x) - f(x-\Delta x)}{\Delta x}\right] \tag{15}$$

or

$$D_1^2(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2} \tag{16}$$

which may be represented by a masking operator of the form

$$[1 \quad -2 \quad 1]$$

The Laplacian is a second-order derivative operator defined as

$$L[f(x,y)] = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{17}$$

The discrete approximation is

$$\begin{aligned} L[f(x,y)] &= D_1^2(x) + D_1^2(y) \\ &= \frac{f(x+\Delta x, y) - 2f(x,y) + (x-\Delta x, y)}{(\Delta x)^2} \\ &\quad + \frac{f(x, y+\Delta y) - 2f(x,y) + f(x, y-\Delta y)}{(\Delta y)^2} \end{aligned} \tag{18}$$

Assuming $\Delta x = \Delta y$, this expression may be written as

$$\begin{aligned} L[f(x,y)] &= (1/\Delta x)^2[f(x+\Delta x, y) + f(x-\Delta x, y) \\ &\quad + f(x, y+\Delta y) + f(x, y-\Delta y) - 4f(x,y)] \end{aligned} \quad (19)$$

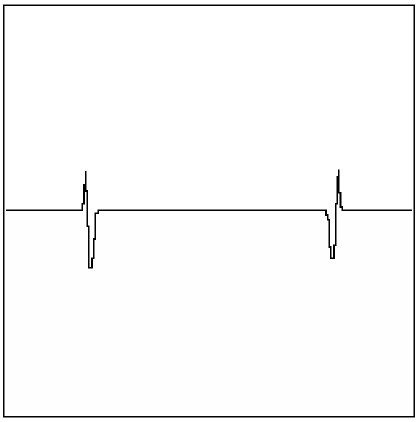
and represented by the operator

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The Laplacian operator is seldom used by itself for edge detection because it is unacceptably sensitive to noise. It is useful in determining whether a given pixel is on the dark side or light side of an edge. It is used in conjunction with a Gaussian filter for edge detection. This is known as a Laplacian of Gaussian (LOG) filter.

Original

After Laplacian (offset by 128)

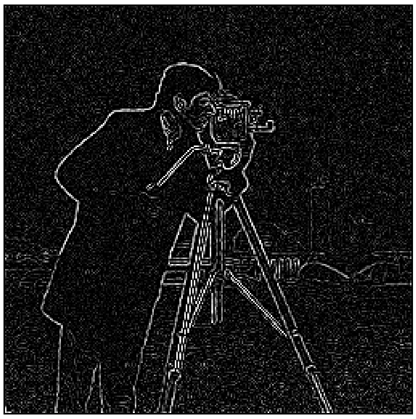


Profile across center

Original

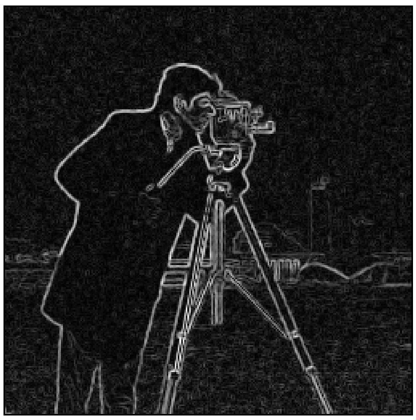
With added noise

Laplacian



Laplacian

Sobel



Sobel

Template Matching

Detecting edges with gradient operators requires only two masks (for the x and y directions). In the template matching approach, up to 12 convolution masks capable of estimating local components of the gradient in the different directions are used. The local edge gradient magnitude is approximated by taking the maximum of the responses for the component masks:

$$G(x, y) = \max\{|G_k(x, y)|\} \quad k = 1, \ldots, n \tag{20}$$

The edge orientation is estimated simply as that of the mask giving rise to the largest value of gradient.

The eight Kirsch edge filter masks are given below:

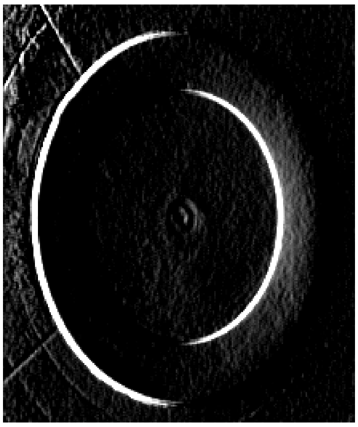
$$\underset{0^\circ}{\begin{bmatrix} -3 & -3 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & 5 \end{bmatrix}} \qquad \underset{45^\circ}{\begin{bmatrix} -3 & 5 & 5 \\ -3 & 0 & 5 \\ -3 & -3 & -3 \end{bmatrix}}$$

$$\underset{90^\circ}{\begin{bmatrix} 5 & 5 & 5 \\ -3 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}} \qquad \underset{135^\circ}{\begin{bmatrix} 5 & 5 & -3 \\ 5 & 0 & -3 \\ -3 & -3 & -3 \end{bmatrix}}$$

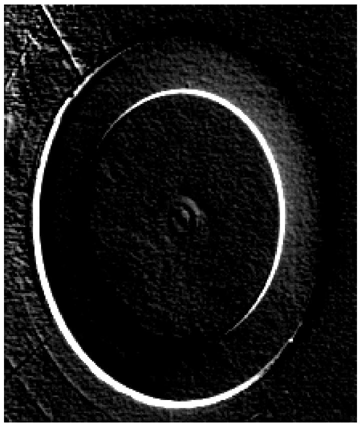
$$\underset{180^\circ}{\begin{bmatrix} 5 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & -3 & -3 \end{bmatrix}} \qquad \underset{225^\circ}{\begin{bmatrix} -3 & -3 & -3 \\ 5 & 0 & -3 \\ 5 & 5 & -3 \end{bmatrix}}$$

$$\underset{270^\circ}{\begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & -3 \\ 5 & 5 & 5 \end{bmatrix}} \qquad \underset{315^\circ}{\begin{bmatrix} -3 & -3 & -3 \\ -3 & 0 & 5 \\ -3 & 5 & 5 \end{bmatrix}}$$

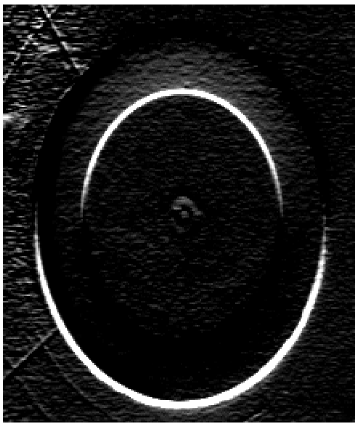
Original



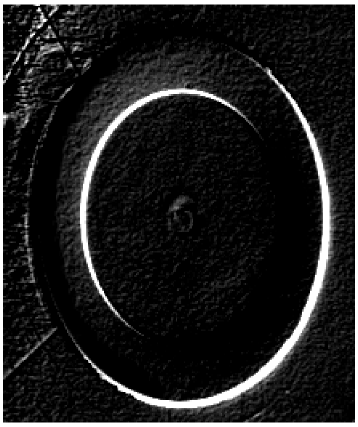
0°



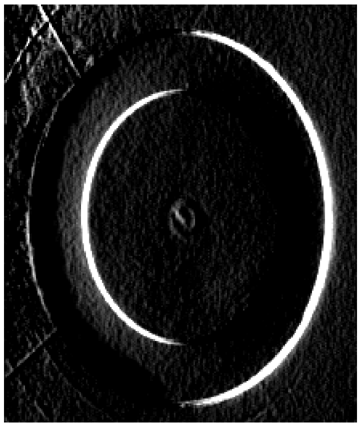
45°



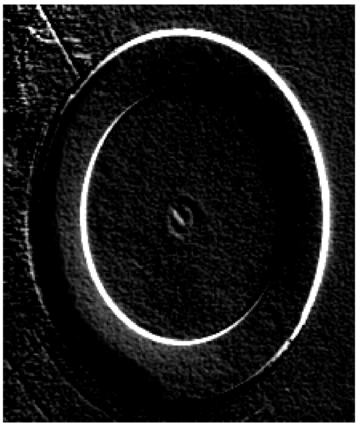
90°



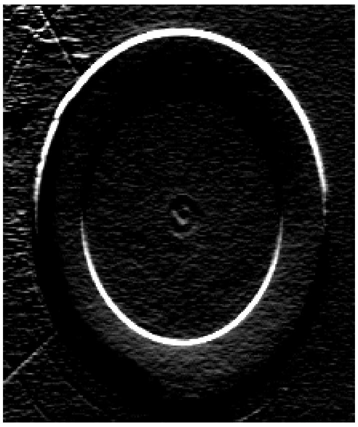
135°



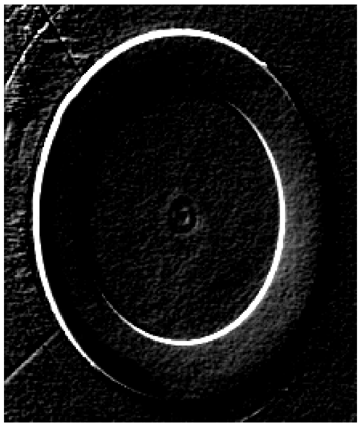
180°



225°



270°



315°