$$
\begin{aligned}
(\Delta x)^2(\Delta s)^2 &= \frac{\int x^2 f f^* \, dx \int s^2 F F^* \, ds}{\int f f^* \, dx \int F F^* \, ds} \\
&= \frac{\int x f . x f^* \, dx \int f' f'^* \, dx}{4\pi^2 \left(\int f f^* \, dx \right)^2} \\
&\geq \frac{\left| \int (x f^* . f' + x f . f'^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2} \\
&= \frac{\left| \int x \frac{d}{dx} (f f^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2} \\
&= \frac{\left| \int f f^* \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2} \\
&= \frac{1}{16\pi^2}.
\end{aligned}
$$

Фильтры Габора

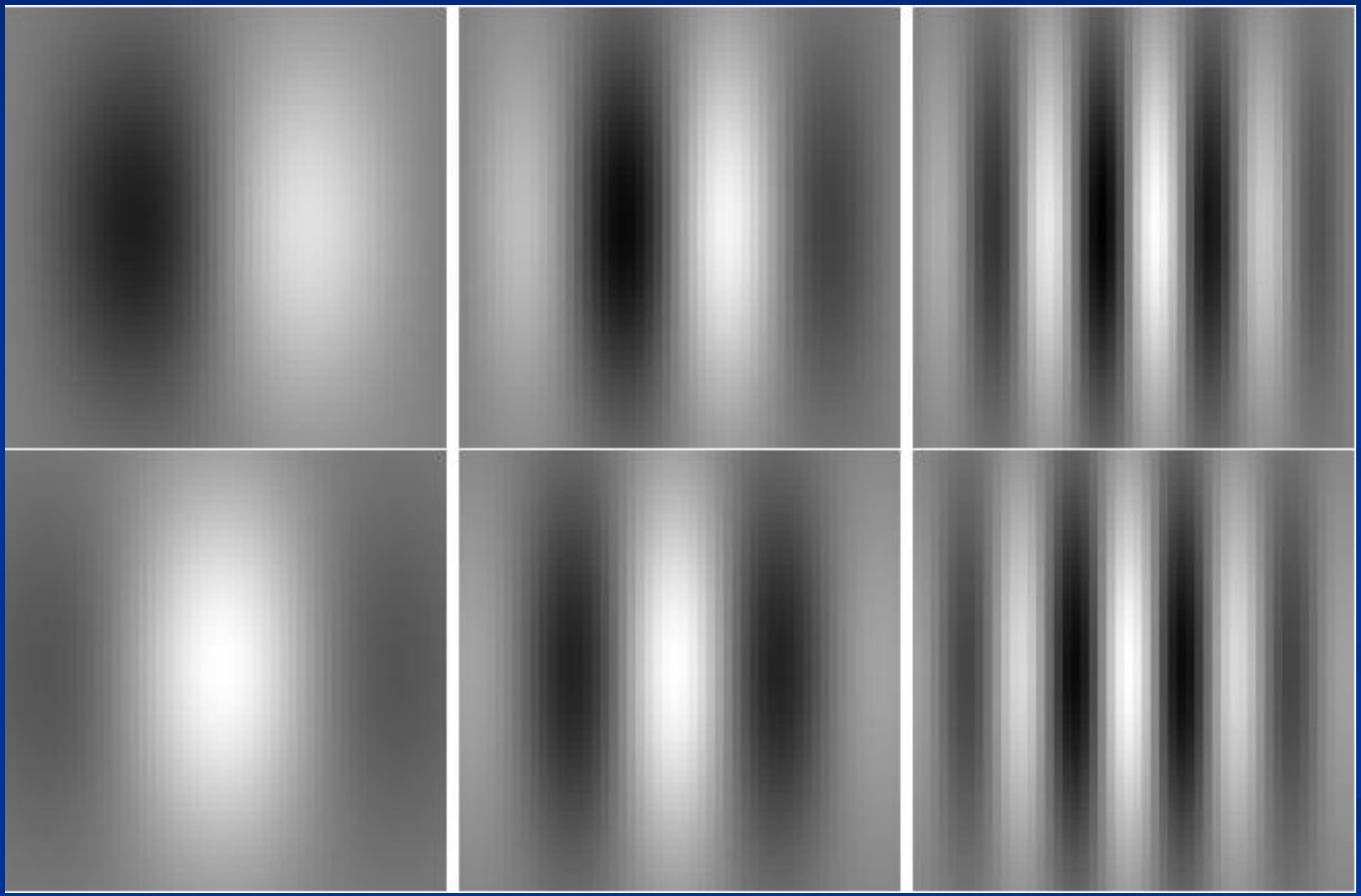
Gabor filters are formed by modulating a complex sinusoid by a Gaussian function:

$$g(x,y) = \overbrace{\frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{\tilde{x}^2}{\sigma_x^2}+\frac{\tilde{y}^2}{\sigma_y^2}\right)\right)}^{\text{gaussian envelope}} \cdot \overbrace{\exp\left(2\pi j\omega\tilde{x}\right)}^{\text{complex sinusoidal}}$$

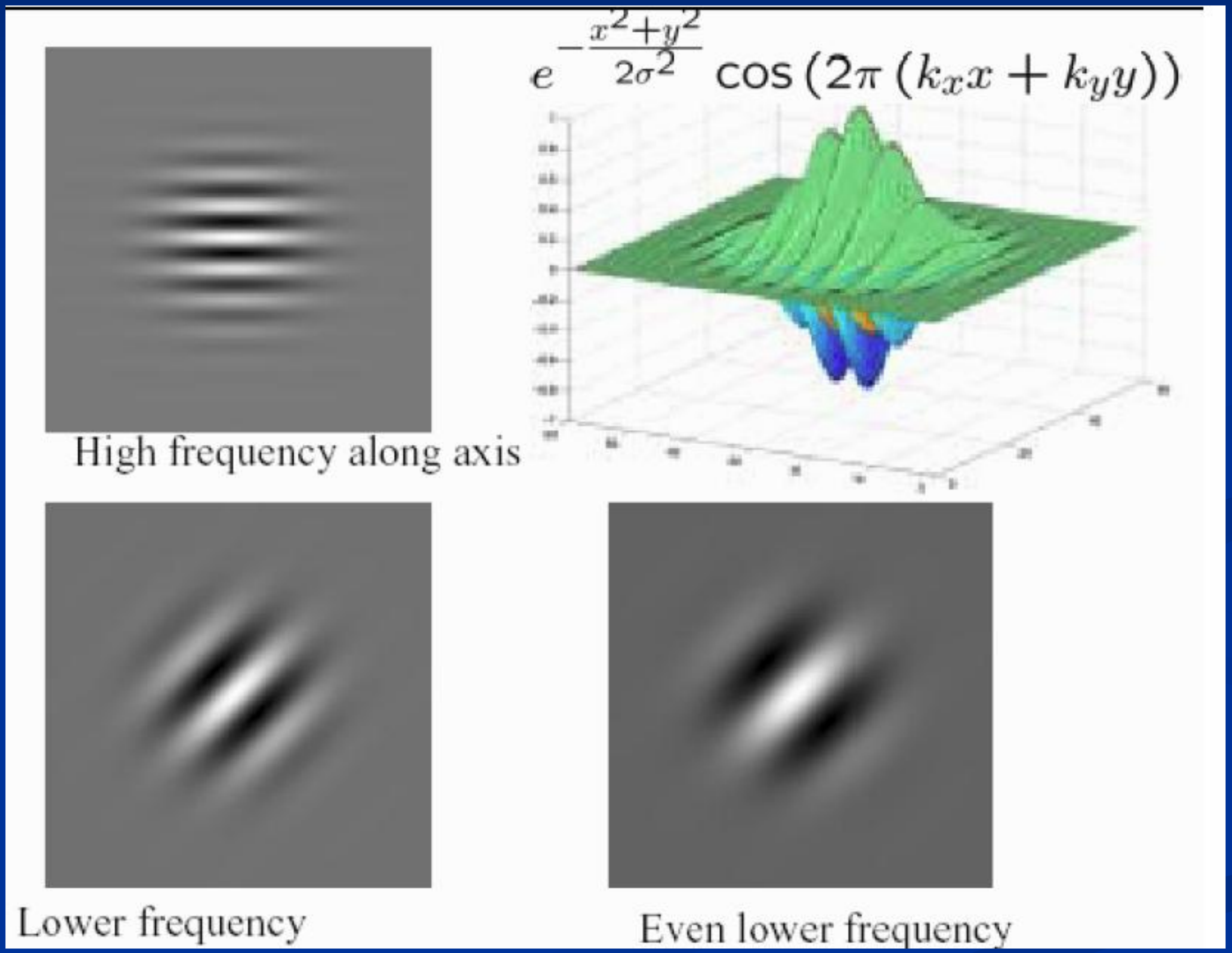
with $\begin{cases} \tilde{x} = x\cos(\theta) + y\sin(\theta) \\ \tilde{y} = -x\sin(\theta) + y\cos(\theta) \end{cases}$

- σ_x and σ_y control spatial extent of filter
- θ is the orientation
- ω is the radial frequency of the sinusoid

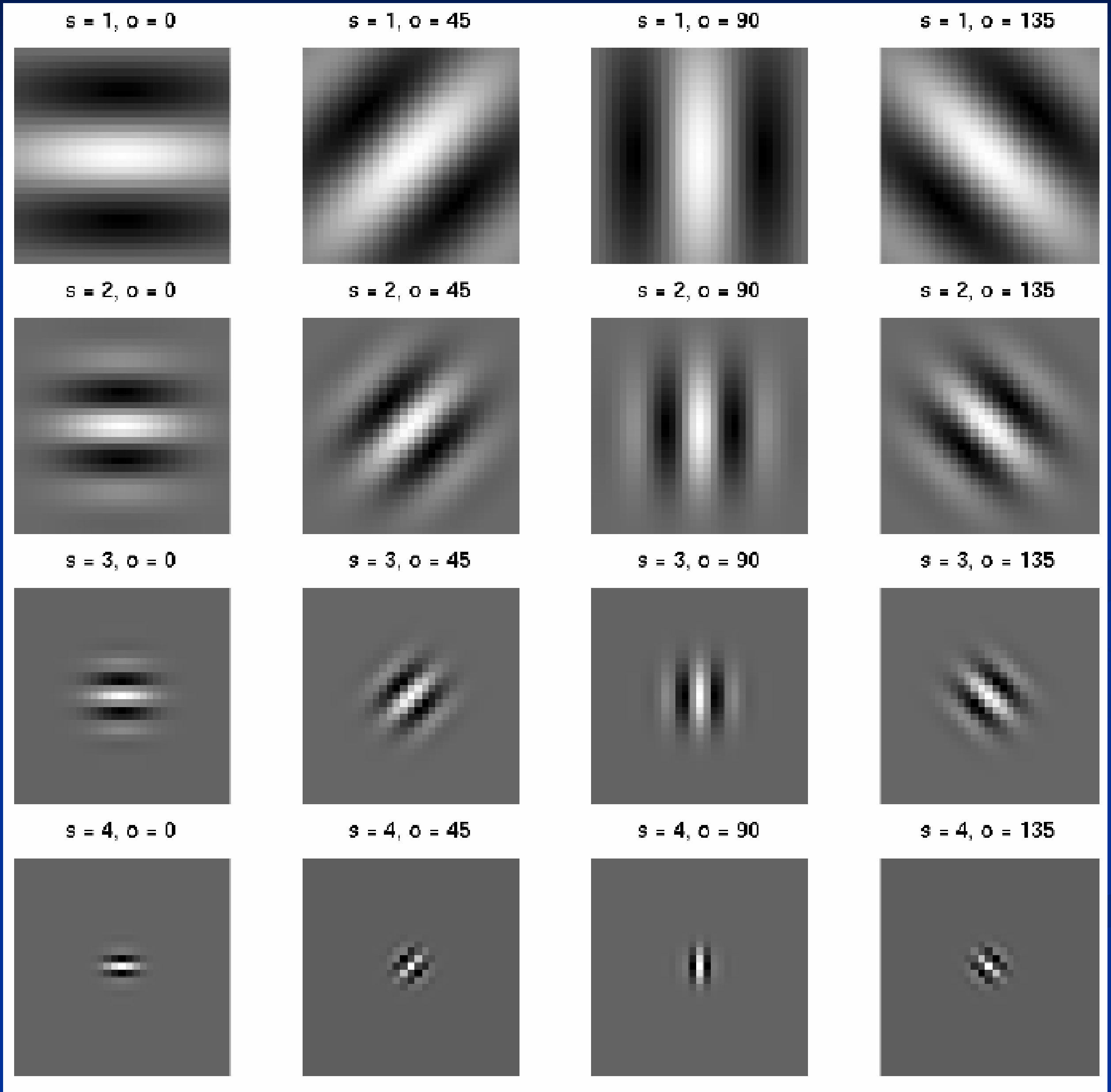
Фильтры Габора



Фильтры Габора



Фильтры Габора



Функции Габора

$$g(x, y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right) + 2\pi jWx\right],$$

$$G(u, v) = \exp\left\{-\frac{1}{2}\left[\frac{(u - W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right]\right\},$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

Габоровские вейвлеты

$$g_{mn}(x, y) = a^{-m} g(x', y'), \quad a > 1, \quad m, n = \text{integer}$$

$$x' = a^{-m}(x \cos\theta + y \sin\theta) \quad \text{and}$$

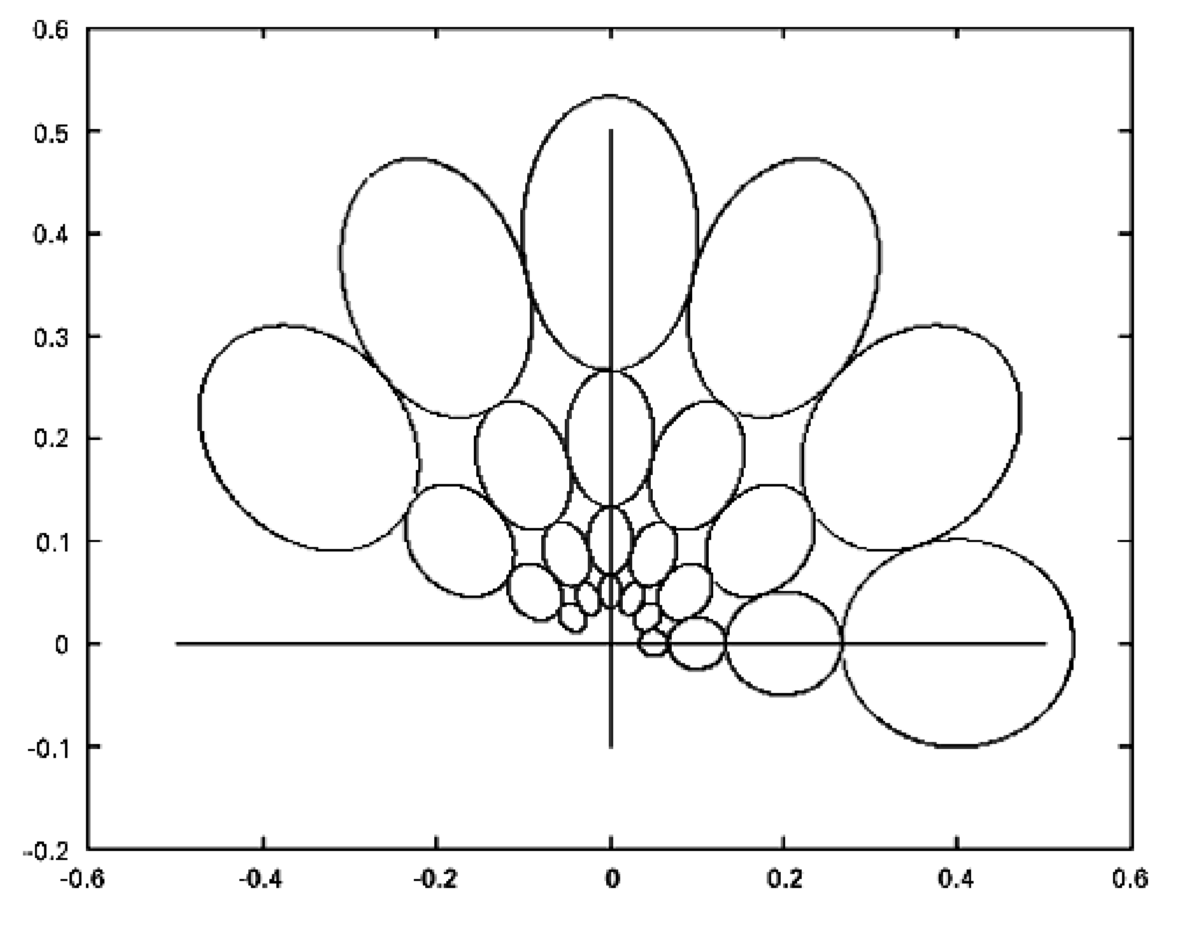
$$y' = a^{-m}(-x \sin\theta + y \cos\theta),$$

$$\theta = n\pi / K$$

K – number of orientations

Габоровские вейвлеты

$$W_{mn}(x, y) = \int I(x, y) g^*_{mn}(x - x_1, y - y_1)\, \mathrm{d}x_1\, \mathrm{d}y_1$$



$U_{\mathrm{h}} = 0.04$, $U_1 = 0.05$, $K = 6$ and $S = 4$

Габоровские вейвлеты

$$a = (U_{\mathbf{h}}/U_{\mathbf{l}})^{1/(S-1)}, \quad \sigma_u = \frac{(a-1)U_{\mathbf{h}}}{(a+1)\sqrt{2\ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right)\left[U_{\mathbf{h}} - 2\ln 2\left(\frac{\sigma_u^2}{U_{\mathbf{h}}}\right)\right] \times \left[2\ln 2 - \frac{(2\ln 2)^2\sigma_u^2}{U_{\mathbf{h}}^2}\right]^{-1/2}$$

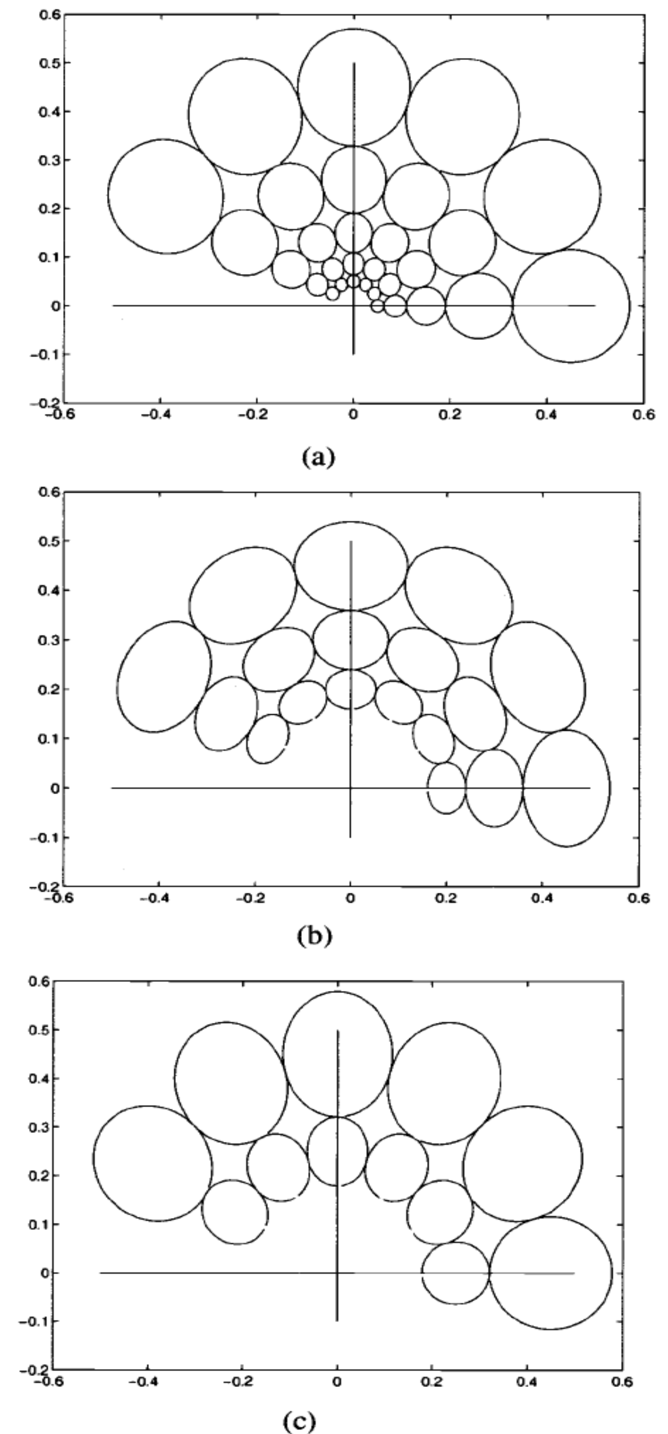
$W = U_{\mathbf{h}}$ and $m = 0, 1, \ldots, S-1$

K=6

$a) \quad \sigma = 5, S = 5$

$b) \quad \sigma = 1.25, S = 3$

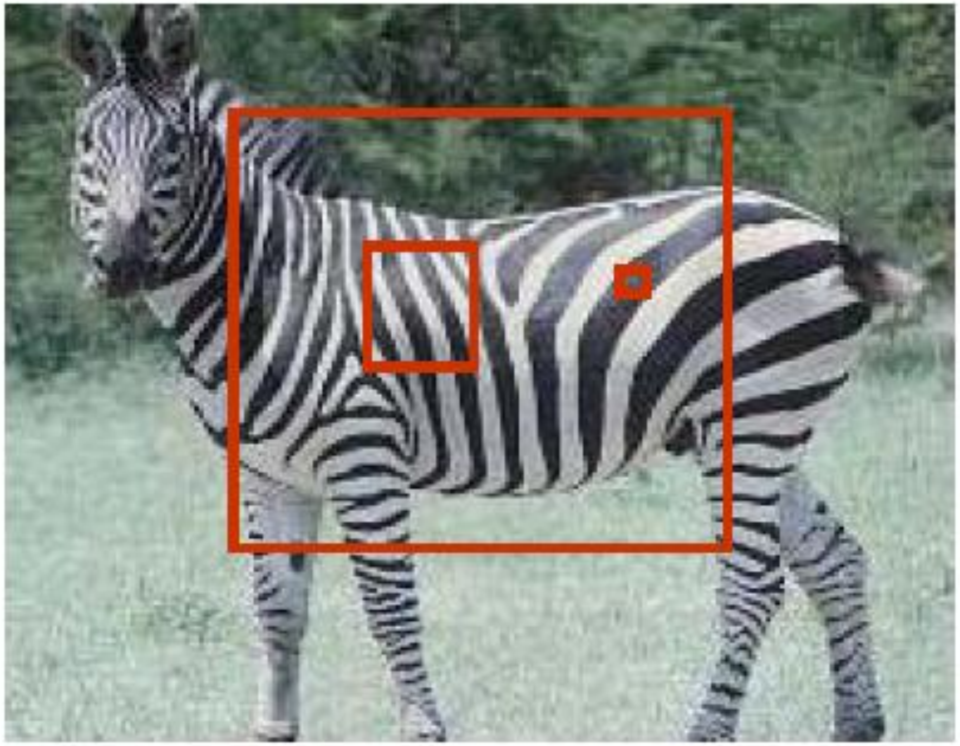
$c) \quad \sigma = 1, S = 2$



Текстуры

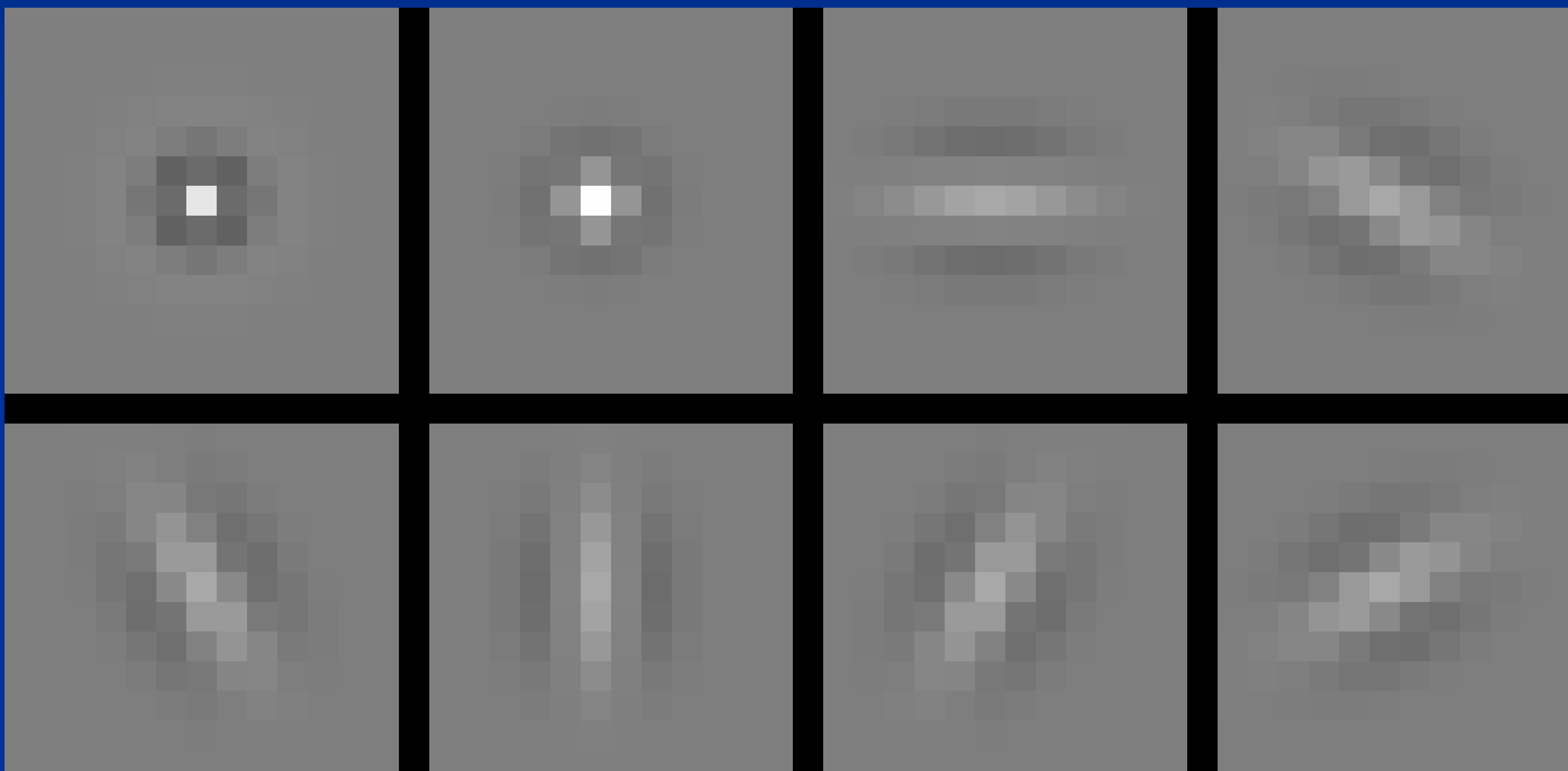
Текстуры

- Whether an effect is a texture or not depends on the scale at which it is viewed.

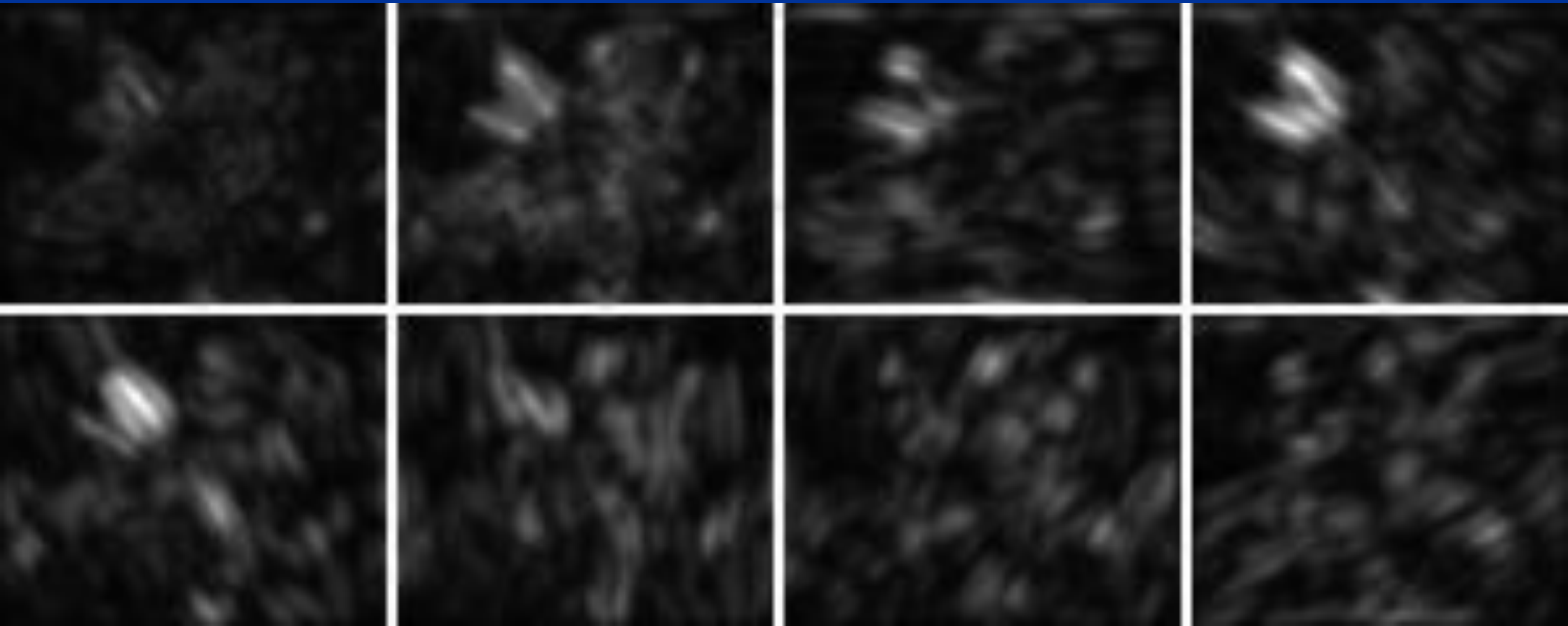


Текстуры

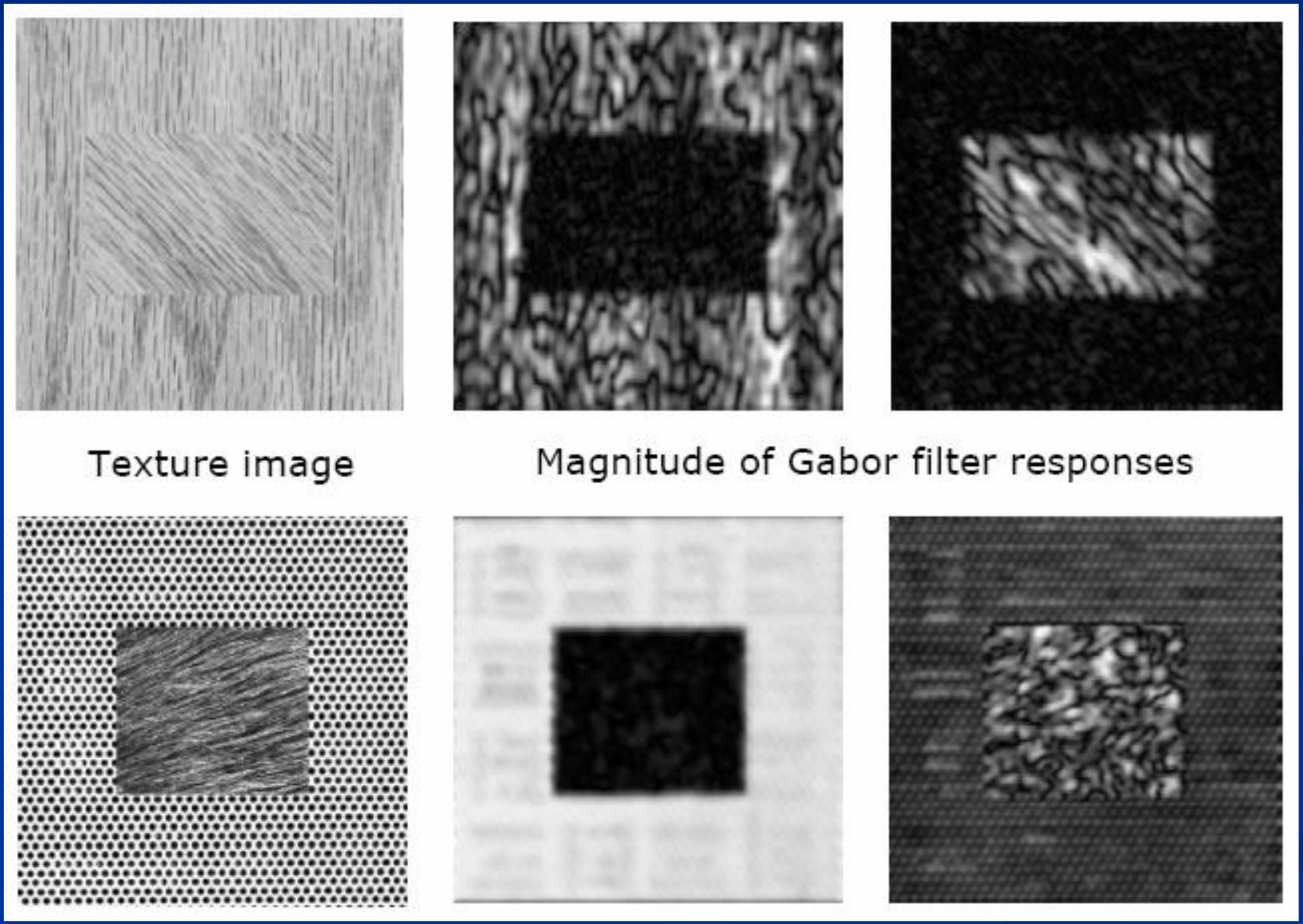
Банки фильтров



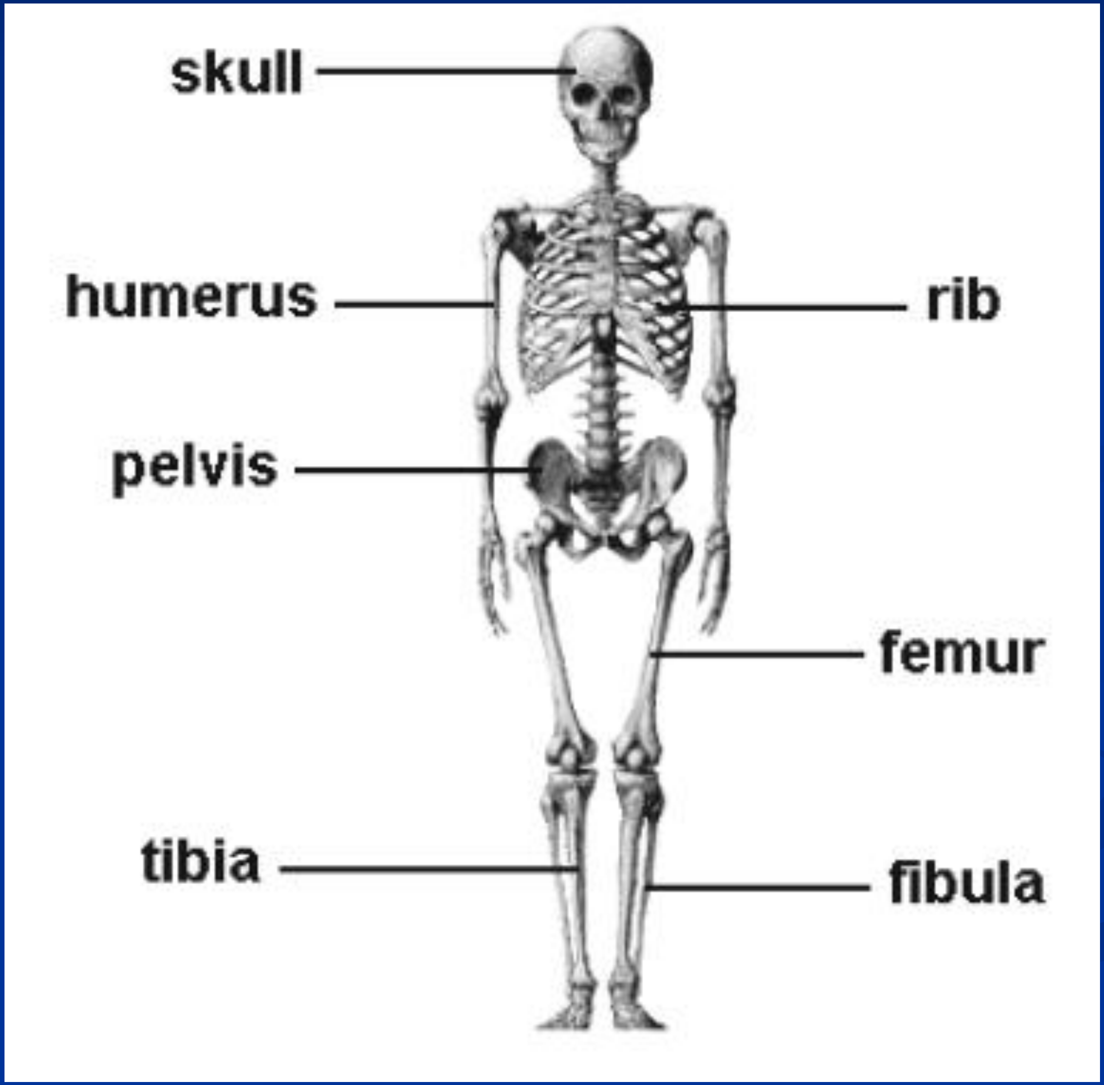
Банки фильтров



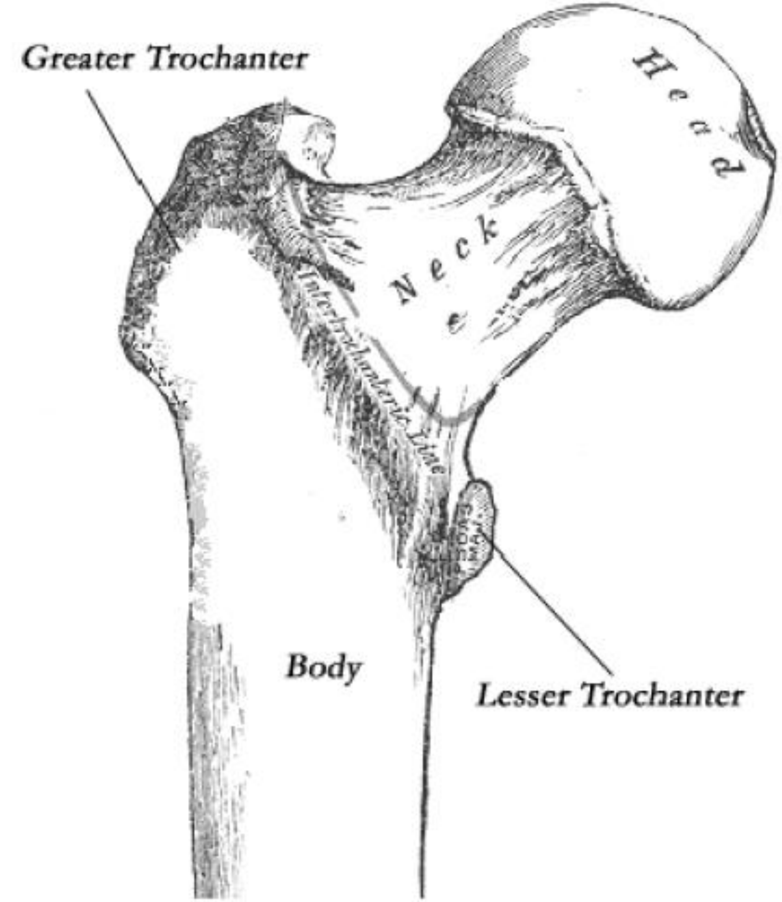
Фильтры Габора

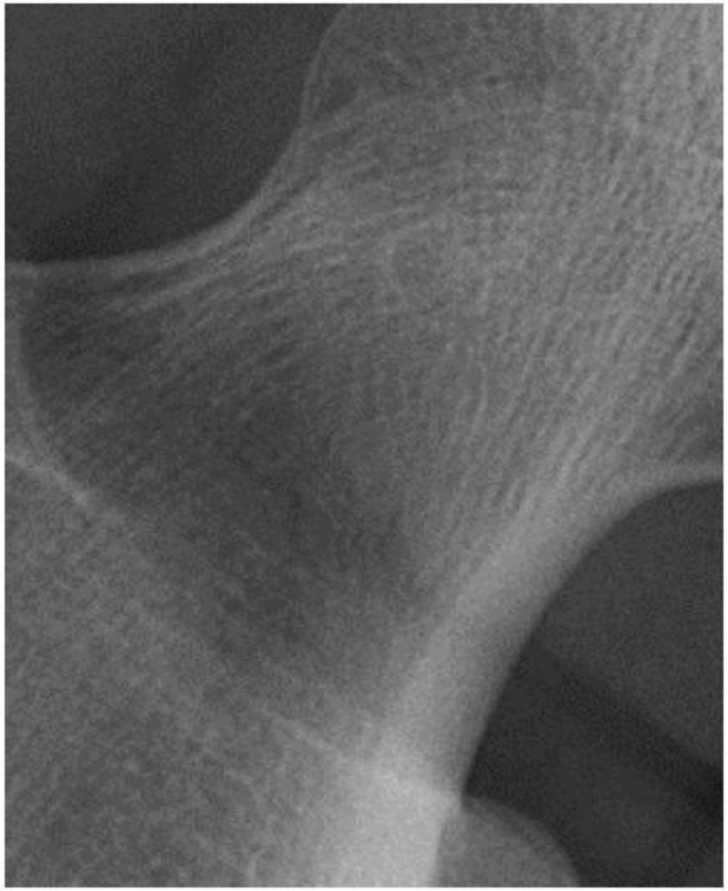


Анализ бедренной кости (femur)



Шейка бедра


Greater Trochanter
Head
Neck
Intertrochanteric Line
Body
Lesser Trochanter



Шейка бедра

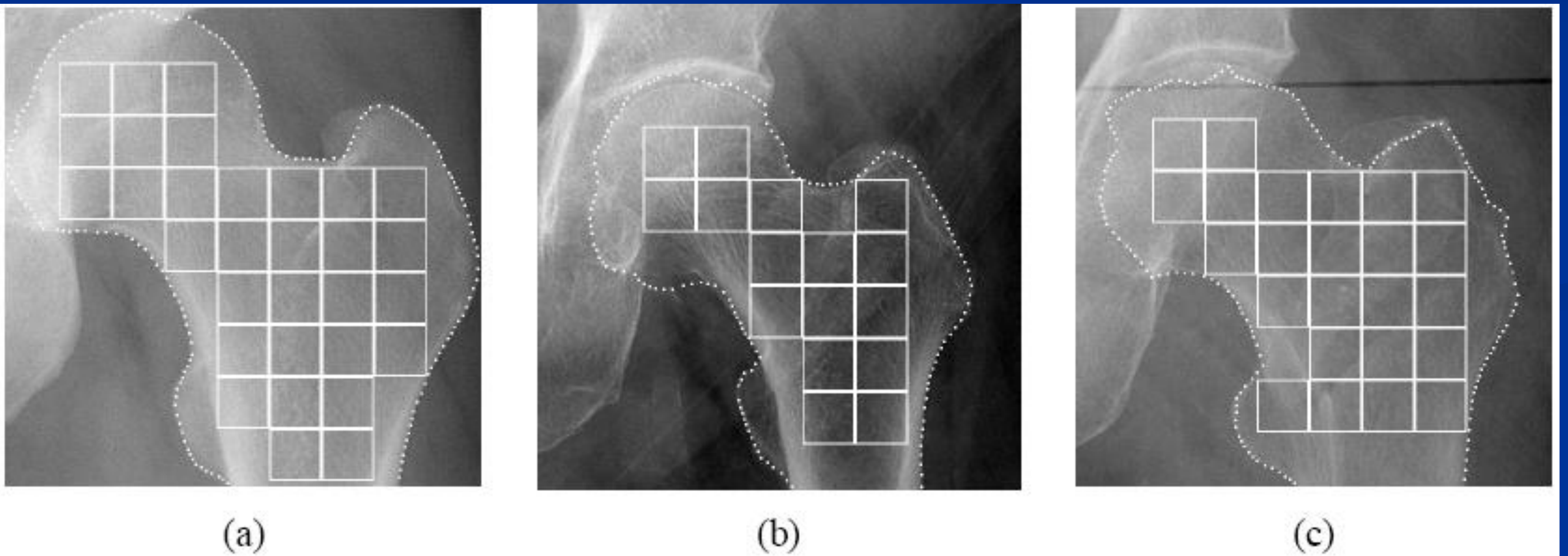


Figure 3.3. A femur is a natural structure that exhibits variations. (a) and (b) above are healthy, while (c) is fractured. Here, (a) is larger than (b). (c) has a relatively shorter neck, due to the fracture. Each grid square is a region sampled for texture orientation feature extraction

$$h(x, y) = g(x', y')\exp(2\pi jfx').$$

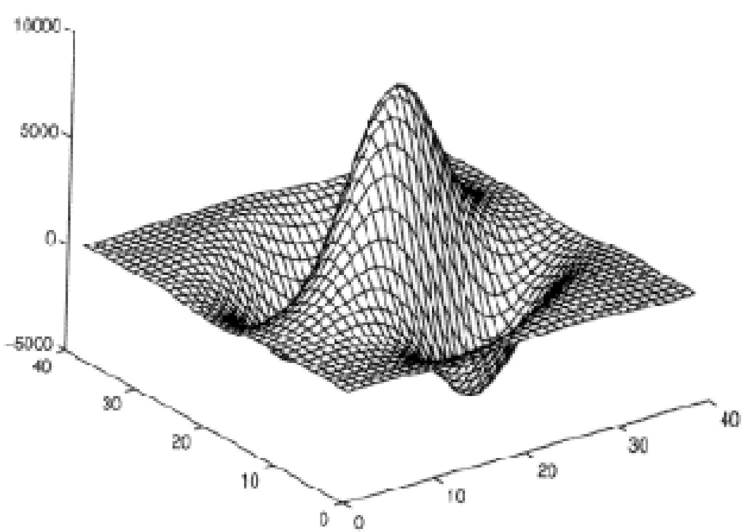
The oriented Gaussian function $g(x', y')$ is given by:

$$g(x', y') = \frac{1}{2\pi\lambda\sigma^2}\exp\left[-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right],$$

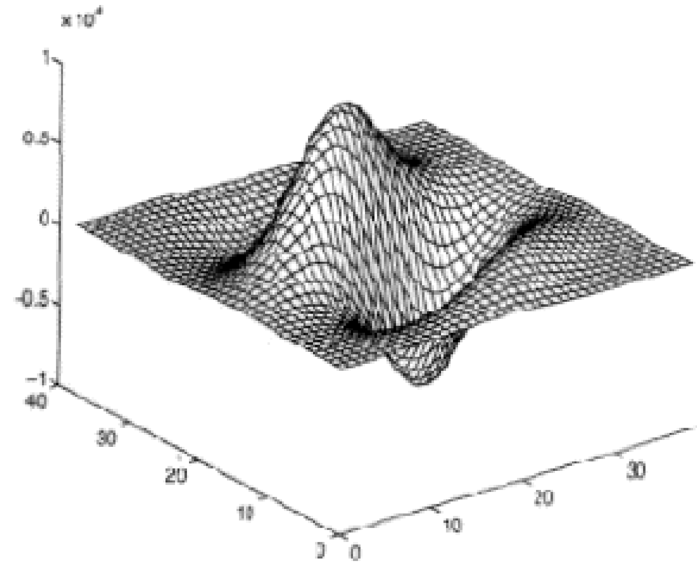
where $(x', y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ are rotated coordinates

$$h_{c,f\theta}(x, y) = g(x', y')\cos(2\pi fx'),$$

$$h_{s,f\theta}(x, y) = g(x', y')\sin(2\pi fx').$$

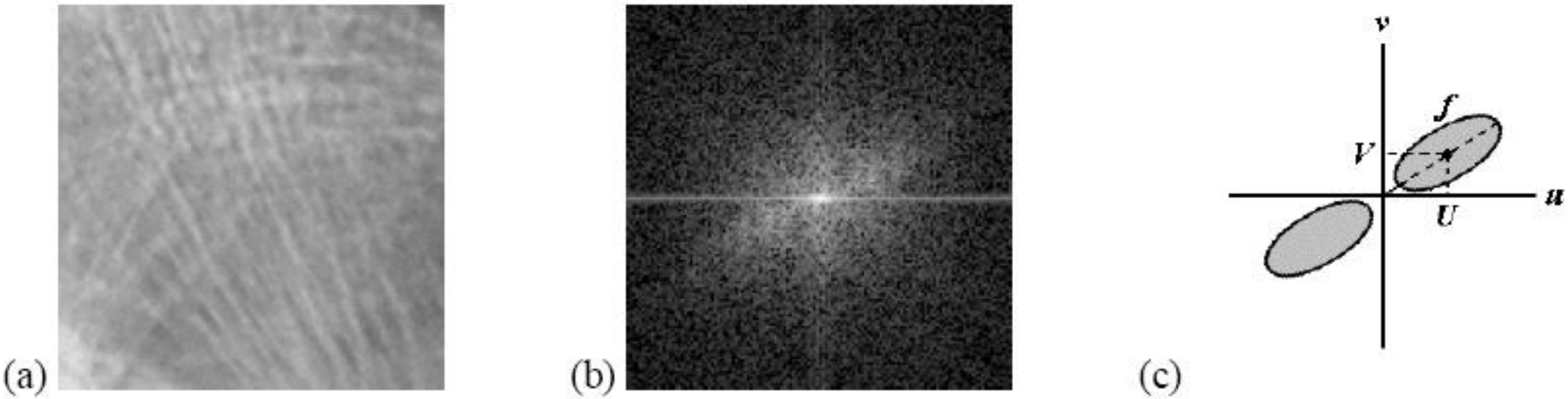


(a)



(b)

Фурье анализ текстуры кости



$$f_p = \sqrt{\overline{U}^2 + \overline{V}^2}$$

$$f_p \cong$$ 0.13 cycles per pixel

A Gabor filter bank of 1 frequency channel and 8 orientation channels is used to extract the orientation of the texture patterns in the femur. The centre frequency f of the Gabor filters is set to 0.13 cycles per pixel, and orientations range from 0° to 157.5°, incrementing in steps of 22.5°.

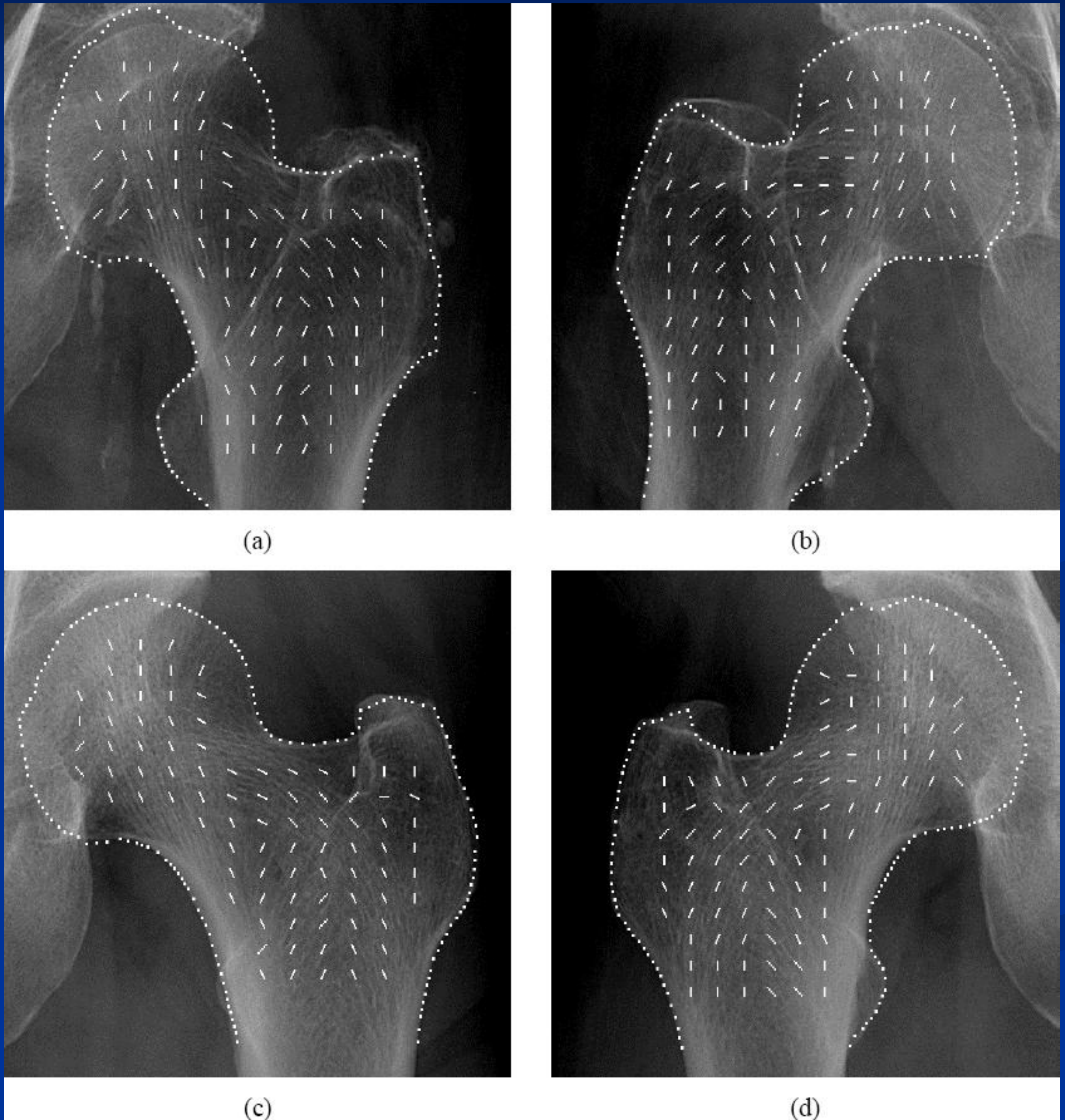
$$e_{c,f\theta}(x,y) = I(x,y) * h_{c,f\theta},$$

$$e_{s,f\theta}(x,y) = I(x,y) * h_{s,f\theta}.$$

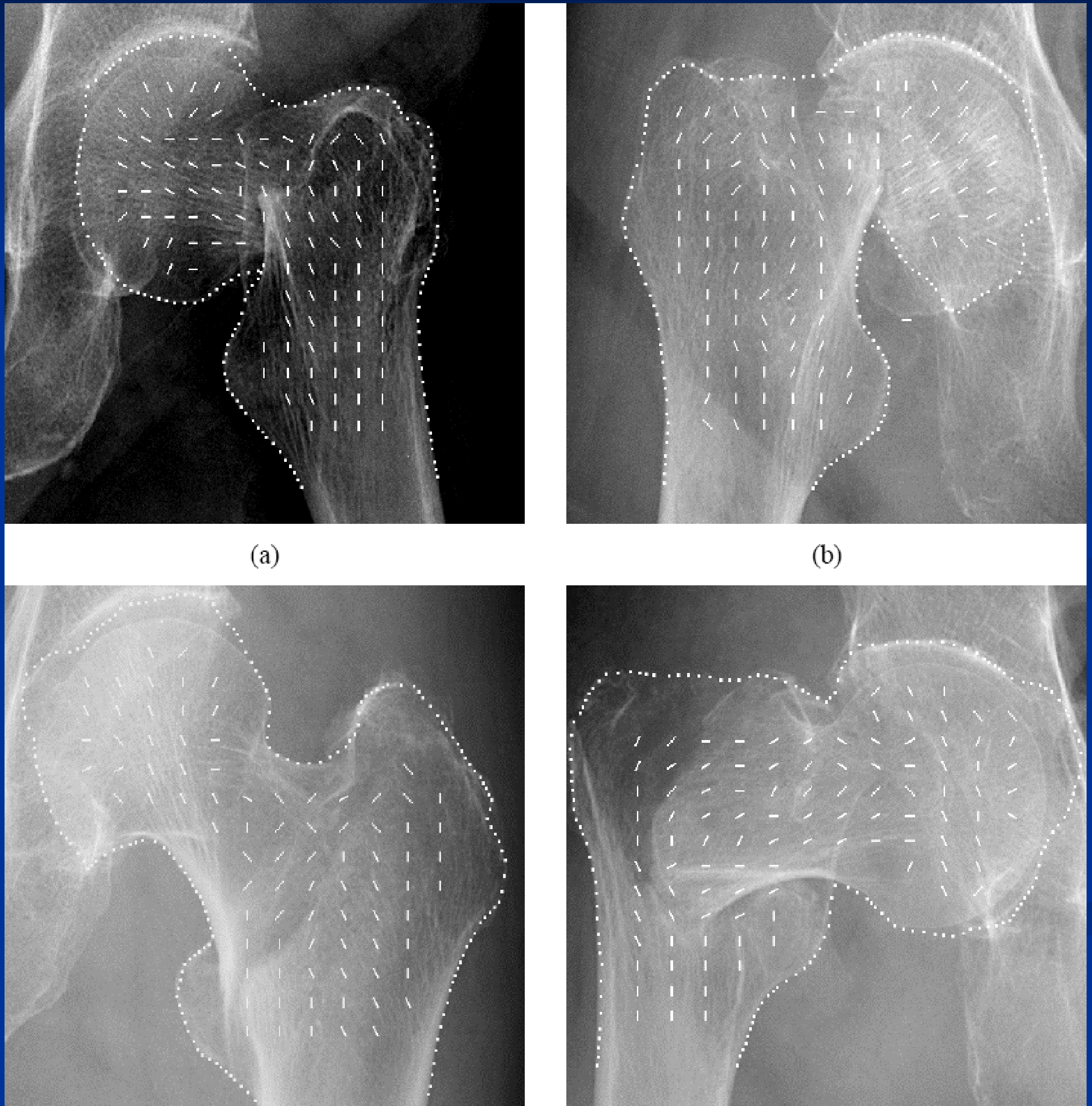
$$E_{f\theta}(x,y) = e_{c,f\theta}^{2}(x,y) + e_{s,f\theta}^{2}(x,y)$$

$$\overline{E}_{f\theta} = \frac{1}{S_x S_y} \sum_{x=1}^{S_x} \sum_{y=1}^{S_y} E_{f\theta}(x,y)$$

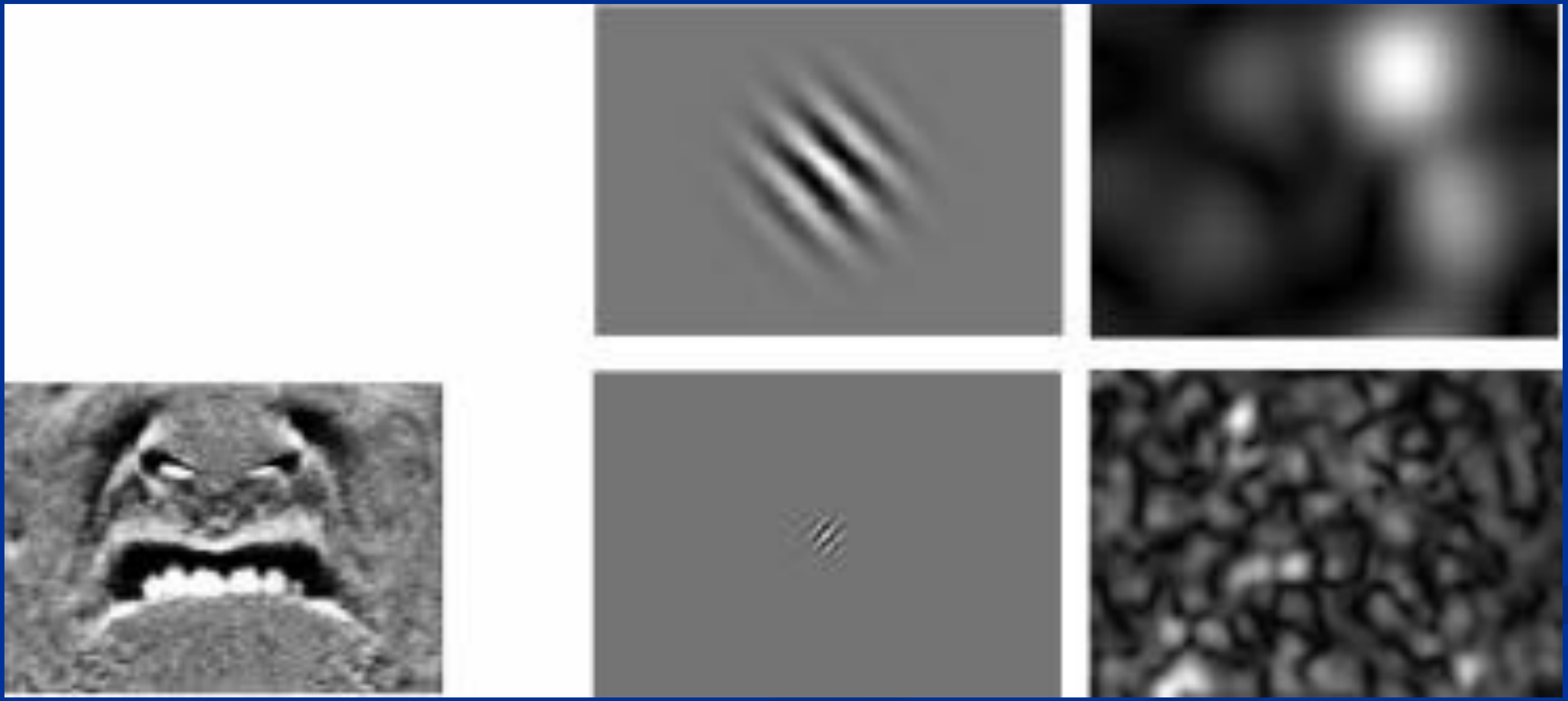
Целые кости



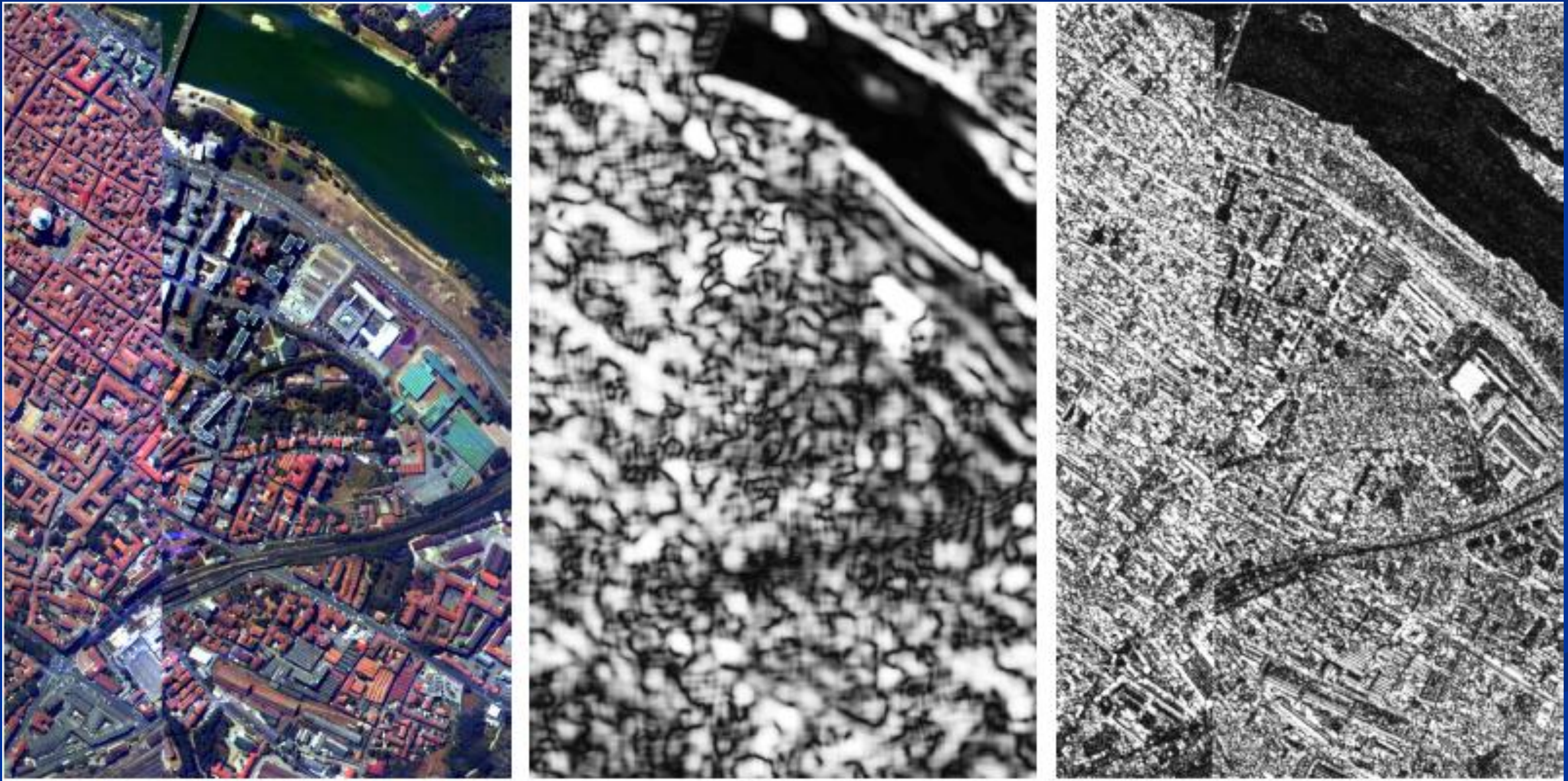
Кости с переломом



Фильтры Габора

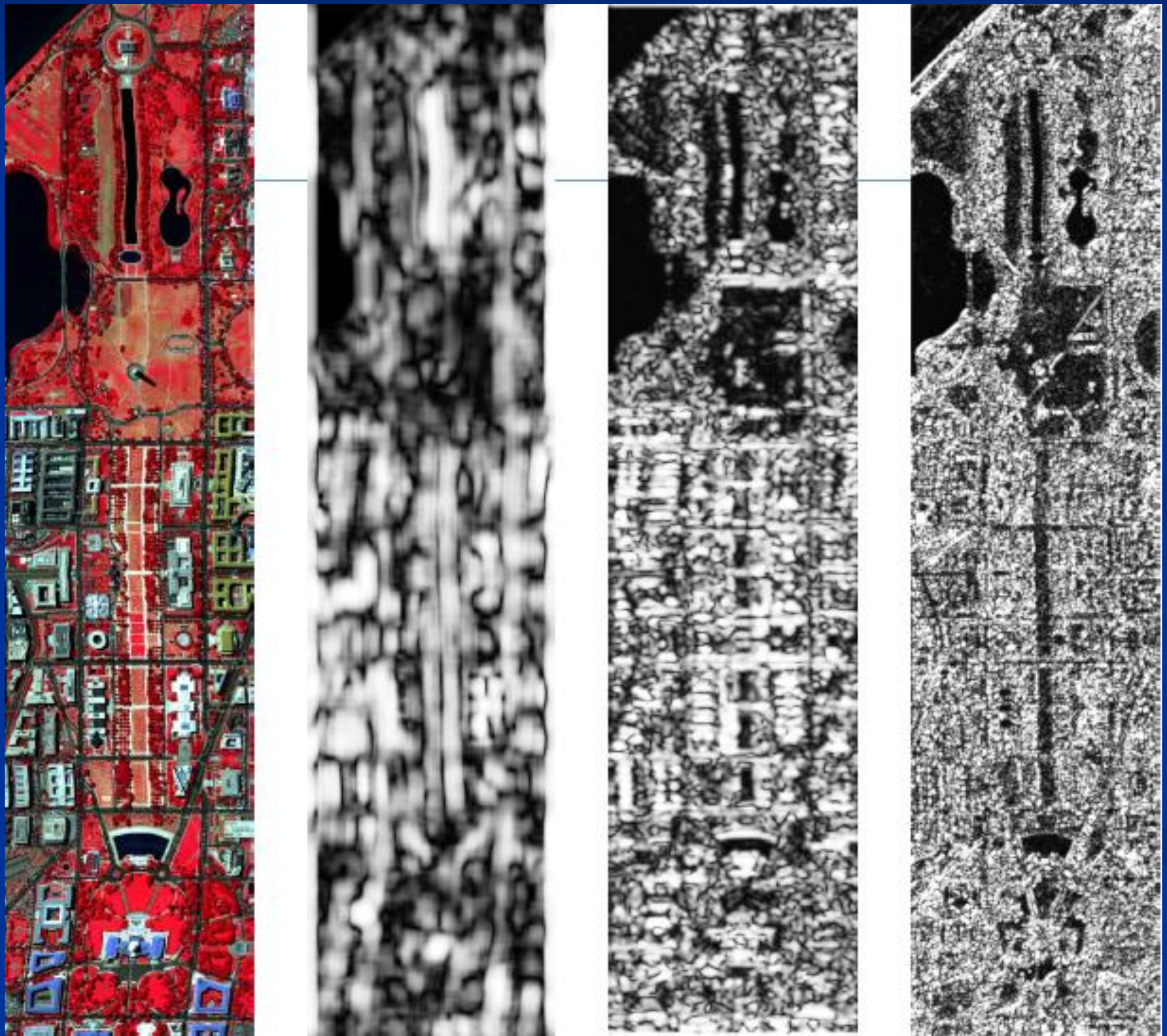


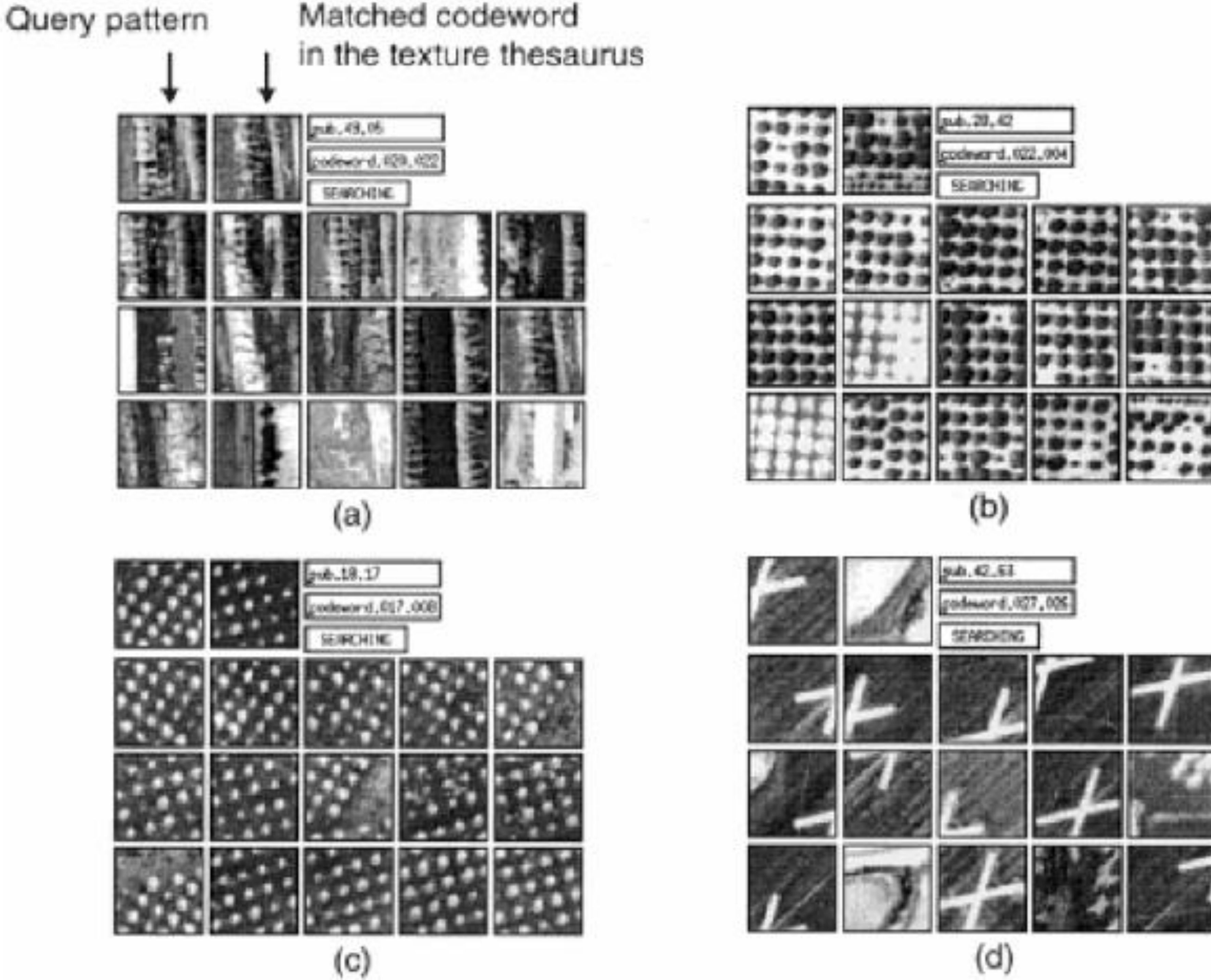
Фильтры Габора

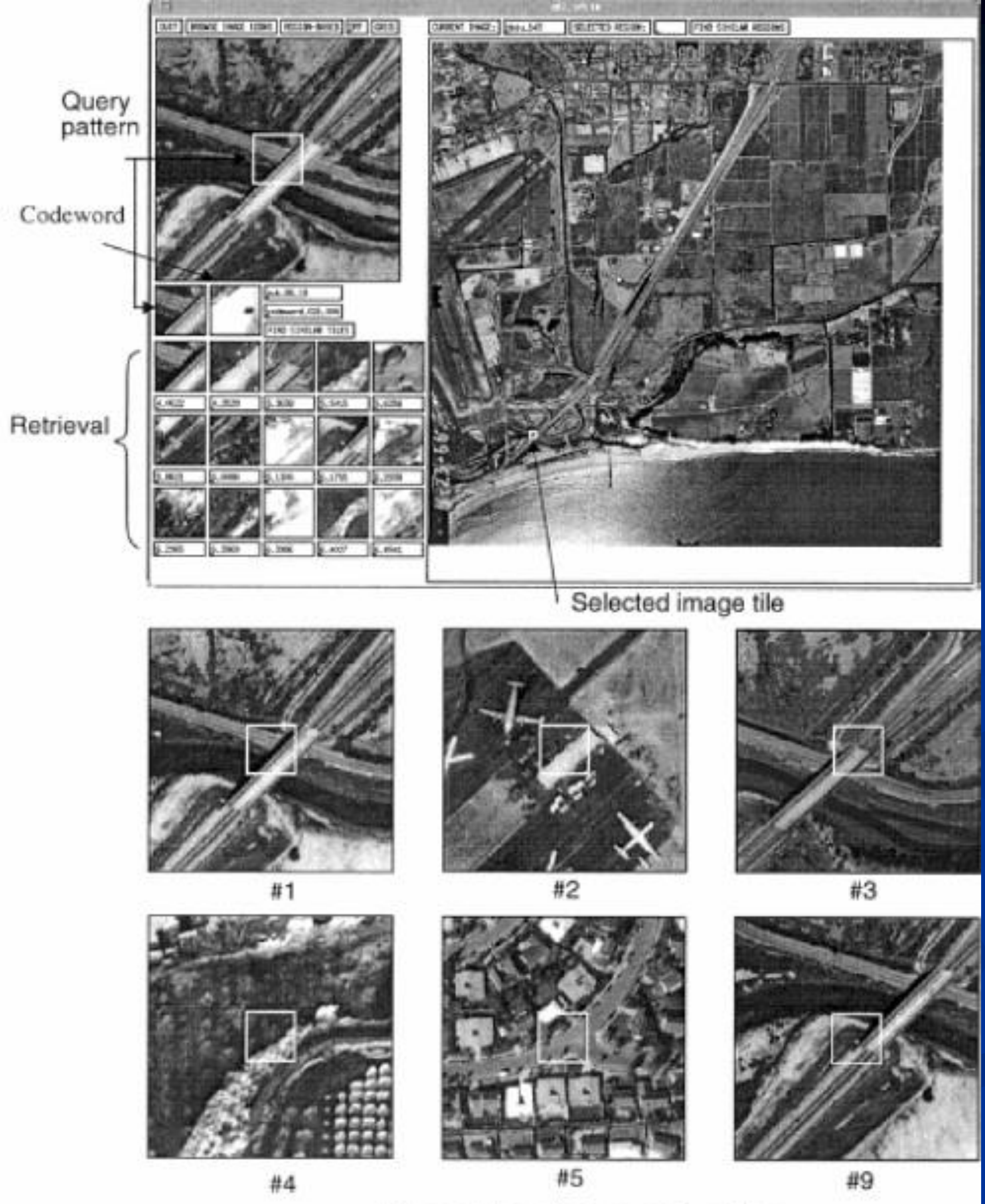


Gabor filter responses for a satellite image.

Фильтры Габора




Query pattern
Matched codeword
in the texture thesaurus
(a)
(b)
(c)
(d)



Full resolution of some retrieved tiles

Теорема. Если функции f и h принадлежат $L^1(\mathbb{R}) \cap L^2(\mathbb{R})$, то

$$\int_{-\infty}^{\infty} f(t)h^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(w)\hat{h}^*(w)\,dw,$$

при $h = f$ из этого следует

$$\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\hat{f}(w)|^2\,dw.$$